

Exercise 23

For the following exercises, determine the domain and range of the quadratic function.

$$f(x) = x^2 + 6x + 4$$

Solution

Any value of x can be plugged into a polynomial function, so the domain is

$$\{x \mid -\infty < x < \infty\}.$$

In order to determine the range, first write the quadratic function in vertex form by completing the square, which makes use of the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = 6$, which means $B = 3$ and $B^2 = 9$. Add and subtract 9 on the right side and then use the identity to make x appear in only one place rather than two.

$$\begin{aligned} f(x) &= x^2 + 6x + 4 \\ &= (x^2 + 6x + 9) + 4 - 9 \\ &= (x + 3)^2 - 5 \end{aligned}$$

Because the coefficient of the squared term is positive, the parabola opens upward; in other words, the squared term takes on values between zero and infinity. The smallest value of $f(x)$ is $0 - 5 = -5$, and the highest value of $f(x)$ is $\infty - 5 = \infty$.

$$\{y \mid -5 \leq y < \infty\}$$

